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Bayesian Characterization and Detection of Rare Binary Signature Events

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LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

Hugh L. Southall

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

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R.C. RAUP
Group 91

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ABSTRACT

A method of imposing a binary classification on a target signature is described that transforms a sequence of signatures into a binary-valued time series using the theory of runs. Under the assumption that the time series can be treated as a set of Bernoulli trials, a Bayesian method of estimating a probability density characterizing the outcome of each trial is considered. The density is used to detect signature events which are found to be rare with respect to the classification imposed on the signatures. Finally, tests of homogeneity are used to partition the observed signatures, when necessary, into equivalence classes having the same density characterization.

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1. INTRODUCTION

Many thousands of Earth-orbiting targets are routinely tracked by ground-based radar and optical sensors. These sensors provide positional measurements of sufficient quality to maintain a catalog of accurate orbital information about the targets, and help keep track of a growing inventory of orbiting payloads and debris. Most of the target signatures (the normalized received-power time series) obtained as a by-product of the tracking operation are currently discarded.

It is interesting to consider if these incidental signatures can be used to obtain additional information about the characteristics of the targets. The principal difficulty in using these data is the lack of any automated analysis tools to reliably process the large number of signatures currently produced. The methods must be simple and broadly applicable due to the large amount of data and the variety of signature and sensor behaviors. This report describes some simple theory for using these incidental signatures to monitor the behavior of targets.

Target signatures obtained during routine tracking of high-Earth-orbit targets by ground-based radars have a duration of at most a few hundred seconds, about the time required for making the precise position measurements used to update the orbital model for the target. For low-Earth-orbit targets the signal-to-noise ratios (SNRs) may be large enough to obtain the required position measurements within a few seconds. In either case, during these tracking times the sensor pointing will not change very much.

Then if the target is three-axis stabilized it will present a nearly constant viewing aspect to the sensor over the duration of the track. For this reason even the signature of a target with a very complex shape can sometimes be characterized as being due to a target with constant, nonfluctuating cross section. The SNR at the sensor receiver will not always be large however, so that the receiver signal associated with a track of such a three-axis stabilized target is best modeled as a stationary random process, generally chi-squared with a number of degrees of freedom determined by the receiver's signal processing algorithms.

If the target exhibits rotational motion with respect to the sensor line of sight (LOS) over the tracking time, however, the average value of the observed signature will generally change. The signature will exhibit trends and other nonrandom fluctuations. This might be the case with orbiting debris, inactive payloads, or spin-stabilized payloads. A notable exception is a sphere-like object whose cross section is not viewing-aspect-angle dependent—it will present a constant cross-section signature regardless of its rotational motion.

Thus a statistical test for a random process applied to the target signature will have different outcomes depending on, among other things, the stability of the target and the viewing geometry of the target/sensor encounter. Such statistical tests have enjoyed a variety of uses in science and engineering. By utilizing one of them each target signature can be tested for randomness, and based on the result each target signature can be classified as being a *stable signature* (passing the test for randomness) or an *unstable signature* (failing the test for randomness).

There are two interesting things about this approach. First, the target signature itself is reduced to a single binary bit of information, i.e., whether the signature collected during the track passes or fails a test of stability. Other independent variables such as time, sensor pointing, and so forth, will prove necessary in utilizing this single bit of information to characterize the target behavior, but the transformation of the entire signature to a single binary bit is as extreme a compression scheme as can be practically imagined. The compression operation somewhat desensitizes the signature processing results to the realities of sensor measurement artifacts and provides greatly reduced database storage requirements.

Second, just as the time series of instantaneous power measurements (which make up the signature of the target) provide one of the most commonly used observables for characterizing the target behavior over the duration of the track, the signature stability test results form a discrete series of binary observables which can be used to characterize the behavior of the target over its orbit, and perhaps its operational life. These signature stability test results will have proven useful if they can be used to fuse, or combine, the information from many tracks into a characterization of the target. To show that this is possible, for each target the signature stability test results will first be used to compute a density function associated with the probability of observing a stable target signature. The probability density is the means of fusing information and characterizing the object.

There are at least two ways to demonstrate the utility of this density function as a characterization of the target. First, if the density function indicates that the probability of observing a stable signature for a given target is quite large, for example, then it would be interesting to know when a relatively rare unstable signature is observed for the object. These rare signatures may hold clues about attitude maneuvering of the target, or might announce a change in target behavior or loss of attitude control. Thus the densities can be used to detect rare events which might reflect characteristics of the target. If these events are rare enough, a substantial reduction in the amount of signature data that must be manually reviewed as part of the process of monitoring a space object's behavior signature is accomplished—an analyst looks mostly at a small number of rare signatures and the process is partially automated.

Second, the probability density may depend on orbital geometry in such a way as to reveal additional clues about object behavior. Some targets are more likely to exhibit unstable signatures when the sun, the target and the sensor are nearly collinear. This is presumably due to attitude control related to alignment of solar cell panels used to power the target, but regardless of the physical reason, it is a statistically demonstrable phenomenon. This behavior has been noted before, at least in [1]. The dependence of the probability density on this and other geometric variables such as the true anomaly and sensor elevation can be quantified as part of the characterization of the normal behavior of the target.

This report collects the theory and illustrates it with a few simple examples. The numerical results include the algorithmic computation of the density function target characterizations and the detection of rare signature events, as well as a demonstration of the statistically significant dependence of the signature stability results on orbital geometry.

Throughout this report classical methods in nonparametric statistics, estimation, and hypothesis testing are utilized. Section 2 describes a method of imposing a binary classification on a target signature to transform a sequence of target signatures into a binary-valued time series. Under the assumption that the time series can be treated as a set of Bernoulli trials, a Bayesian method of estimating a probability density characterizing the outcome of each trial is considered in Section 3. In Section 4, the density is used to detect signature events which are found to be rare with respect to the classification imposed on the signatures. Finally, in Section 5 tests of homogeneity are used to partition the observed signatures, when necessary, into equivalence classes having the same density characterization.

2. BINARY CLASSIFICATION OF SIGNATURES

Motivated by the discussion of Section 1, a test for randomness is proposed as the criterion for classification of target signatures because, under the assumed observation conditions, a three-axis stabilized target will usually produce a mathematically random signature. Consequently, the terms "random signature" and "stable signature" will be used interchangeably. Notable exceptions to this rule are the sphere-like target with a cross section which does not depend on viewing aspect angle and the undersampled target with a fluctuating cross section having a bandwidth which is not small compared to the sensor sampling bandwidth. These targets will often have a mathematically random signature regardless of the true stability of the target or the observation conditions.

It will be argued in Section 5 that a test of homogeneity will also be needed to interpret the results of the signature classification. Although a number of distinct implementations of tests of randomness and homogeneity are available, the theory of runs can be applied to both problems [2]. In order to avoid the introduction of two separate distribution theories, the theory of runs will be used to implement both the test for randomness (needed to classify the target signatures in this section) and the test of homogeneity (needed in Section 5). This theory is therefore recalled here and only referenced in Section 5.

The theory of runs is well known in nonparametric statistics, so a detailed discussion is not needed here. In this report a random signature is defined to be a signature that can be transformed into a sequence of Bernoulli trials by thresholding—the probability that any sample of the signature falls above a threshold is a constant value P_γ, independent of the value of any other sample and dependent on the value of the threshold γ [3]. In Figure 2-1, a threshold value has been set in order to compute the number of runs.

The number of signature samples above the threshold is denoted by n_1 and the number below by n_2. The value of the threshold is not critical. Often the sample median value of the data set is used, so that $n_1 = n_2$. The runs statistic R is defined to be one plus the number of times that the time series crosses (either while increasing or decreasing) the threshold value γ. The density of R under the hypothesis of randomness is derived by a combinatorial argument such as the ones found in [4] or [5] and is given by

$$f_R(r) = \frac{\binom{n_1 - 1}{(r-1)/2}\binom{n_2 - 1}{(r-3)/2} + \binom{n_1 - 1}{(r-3)/2}\binom{n_2 - 1}{(r-1)/2}}{\binom{n_1 + n_2}{n_1}},$$

when r is odd and

$$f_R(r) = \frac{2\binom{n_1 - 1}{r/2 - 1}\binom{n_2 - 1}{r/2 - 1}}{\binom{n_1 + n_2}{n_1}}. \tag{2.1}$$

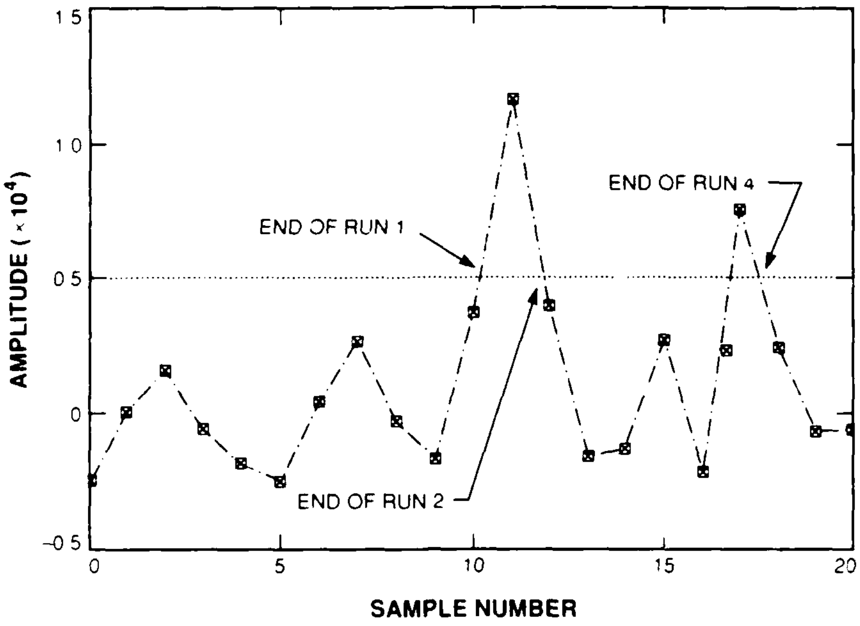

Figure 2-1. A two-tailed test for randomness based on the theory of runs is used to classify the signature.

when r is even.

If the observed number of runs is either too large or too small then the hypothesis of randomness is rejected. A two-tailed test based on the distribution of Equation (2.1) tests the hypothesis of randomness by fixing the desired probability of erroneously classifying a random signature as nonrandom (called a type I error). Two thresholds are set, and if the number of observed runs R is above the larger or below the smaller threshold, then the hypothesis of randomness is rejected.

The thresholds are generally set such that they determine two rejection regions of approximately equal probability under the assumption of randomness. The sum of these probabilities is set to the desired probability of committing a type I error. If the distribution under an alternative hypothesis can be formulated, then the probability of erroneously classifying a nonrandom signature as random can be set (called a type II error). This is done, for example, in [5] using a Markov model as the alternative hypothesis. There is no physical reason, however, to formulate such an alternative hypothesis in this application—it is satisfactory to accept the hypothesis of randomness with a specified probability of erroneously classifying a random signature as nonrandom.

The number of signature samples, to be useful, is generally large enough that the distribution of Equation (2.1) has a Gaussian envelope. In [4] the mean $\mathbf{E}[R]$ and variance $\mathbf{V}[R]$ of R are determined to be

$$\mathbf{E}(R) = 1 + \frac{2n_1n_2}{n},$$

and

$$\mathbf{V}(R) = \frac{2n_1n_2(2n_1n_2 - n)}{n^2(n-1)} \tag{2.2}$$

with $n = n_1 + n_2$ so that when both n_1 and n_2 are greater than, say, 10 the statistic

$$z = \frac{R - \mathbf{E}(R)}{\sqrt{\mathbf{V}(R)}} \tag{2.3}$$

can be treated as normal, avoiding evaluation of Equation (2.1).

Examples of both a stable and an unstable signature as determined by the test for randomness with a probability of a type I error equal to 10^{-3} are shown in Figures 2-2 and 2-3.

Mathematically, the distinction between a stable and an unstable signature is determined by the theory of runs. But intuitively the unstable signature will have an average value that tends to change, a phenomenon called a trend. The stable signature will have an average value that does not have a significant variation. In each figure a small area of the complicated signature structure is expanded to show an average value suggested by a cubic polynomial fit. The normal approximation to Equation (2.1) was actually used to implement the test, so the statistic (Equation (2.3)) was tested against the normal distribution.

The procedure specified in this section allows each target signature to be tested and declared either stable or unstable with a controlled probability of erroneously declaring that a stable signature is unstable. The results of these tests can be used to characterize the probability that a target signature will be stable, assuming no change in target behavior. This will be made more precise in the next section.

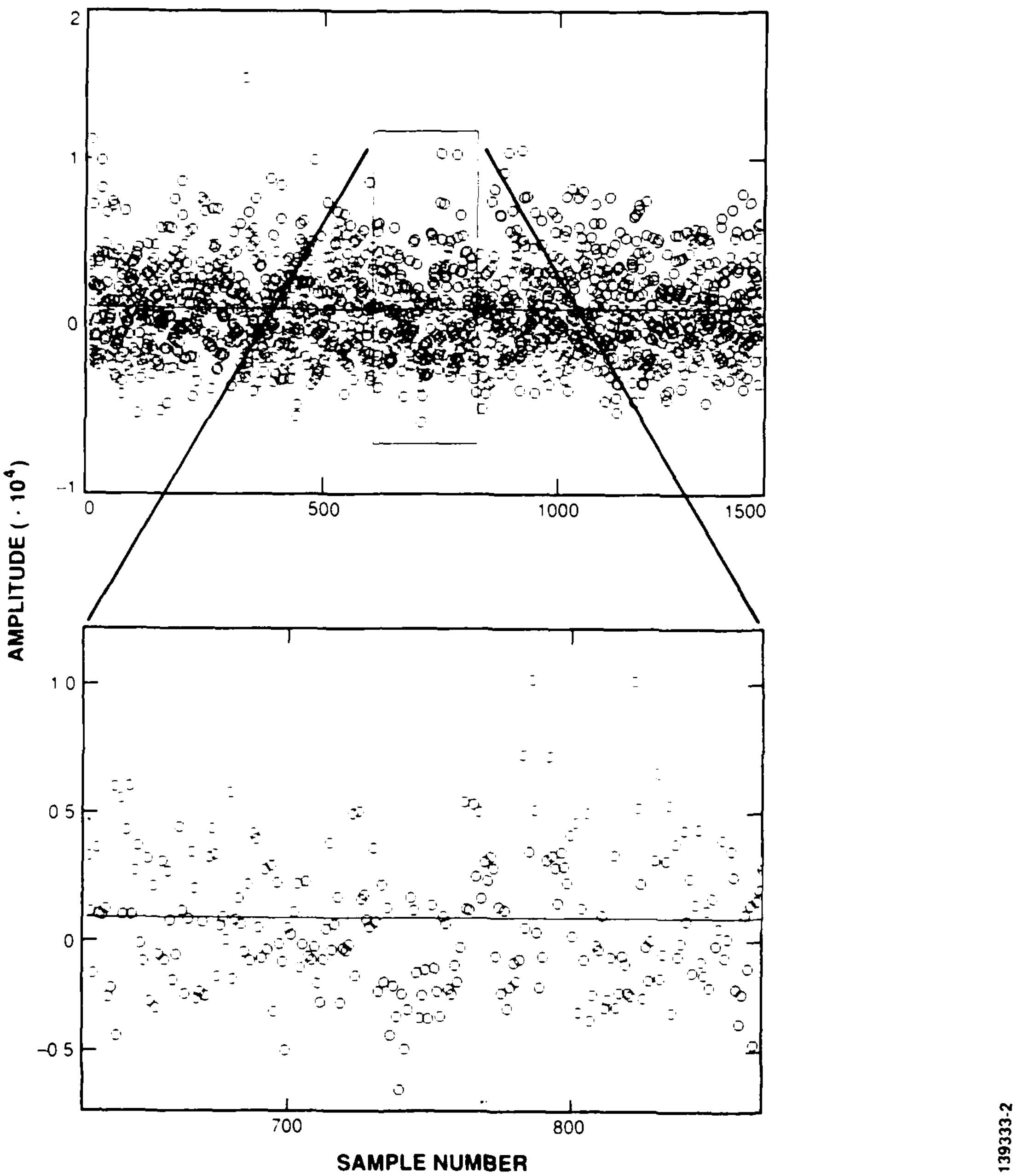

Figure 2-2. Stable signatures, such as this one determined by the runs test, tend to have an average value that does not change. The fluctuations are due to measurement noise, not target cross-section variation.

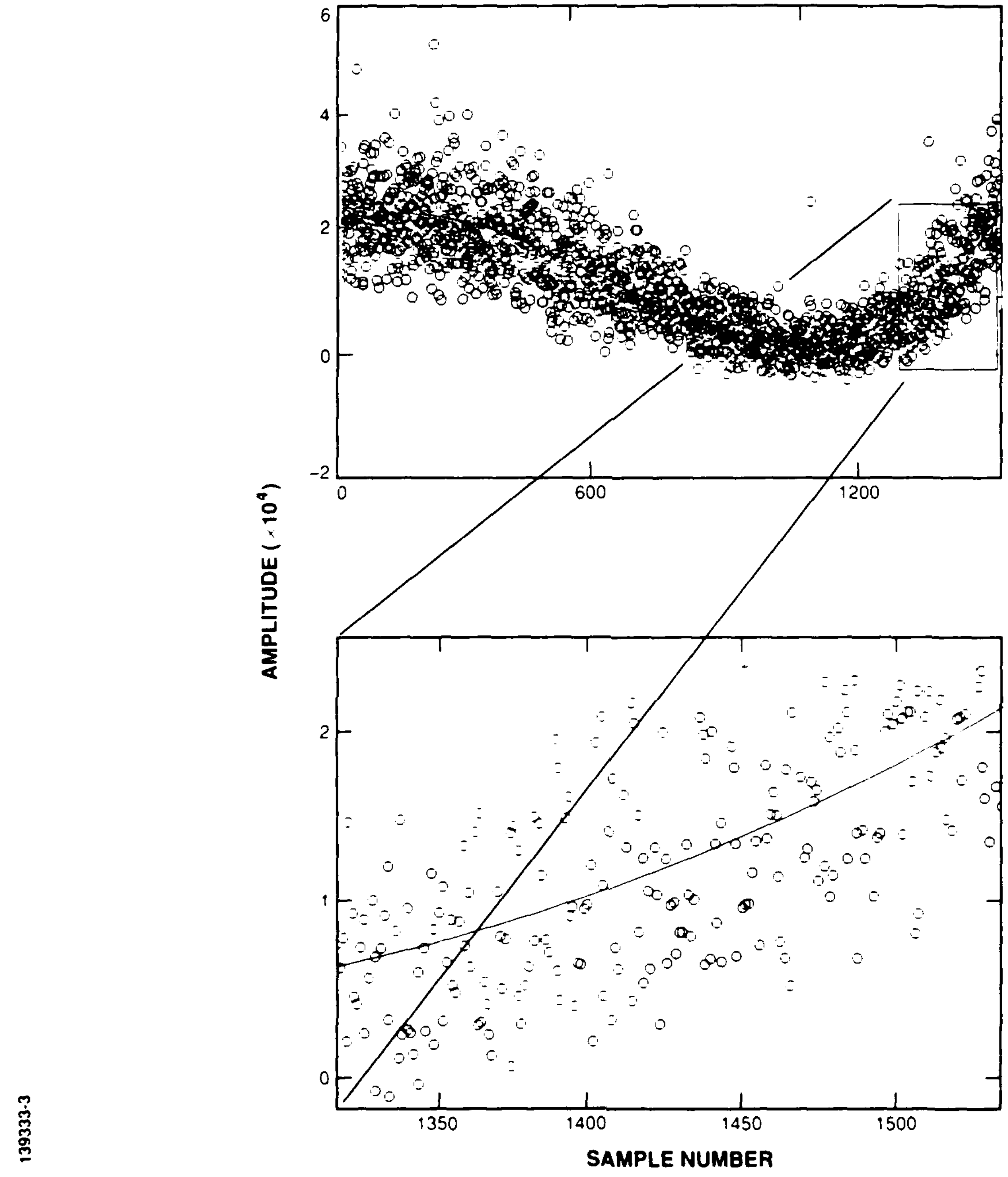

Figure 2-3. Unstable signatures, such as this one determined by the runs test, tend to have a changing average value. The trend is due to target cross-section variation caused by a change in viewing aspect and the complex shape of the target. The fluctuations around the trend are due to measurement noise.

3. PROBABILITY DENSITY CHARACTERIZATION

Section 2 described a test for stable signatures which implements a binary classification of each target signature. In order to discuss this in the language of experiments having binary outcomes the value *true* will be associated with each signature which passes the test for stability, and the *value* false will be associated with each signature that fails the test for stability. Each test of a signature will be referred to as a "trial." Ultimately, it is desirable to use a data base of trial results on a particular target to characterize the probability of declaring a signature to be stable on the next occasion that the target is tracked, under the assumption that the target has not changed its characteristic behavior. The development of such a characterization in this section is the next step in the evolution of the methodology which will be continued in Sections 4 and 5.

Specifically, it is assumed that the results of a sequence of target signature tests on a given target, *which has not altered its characteristic behavior*, can be described by a Bernoulli trial [3]. By assumption, the outcome of each trial is independent of the outcome of any other trial. In addition it is assumed that each trial will have an outcome of true with unknown probability T. The Bernoulli trial first appeared in the test for stable signatures of Section 2 in conjunction with the theory of runs.

The value of the unknown probability T is clearly restricted to the interval $0 \leq T \leq 1$. To incorporate such an interval constraint, suppose that T is itself a random variable and derive all attributes of T from the conditional density $p_{T|\Theta}(t|\theta)$, which will be chosen from a class of densities with support restricted to the closed interval $[0, 1]$. The random variable Θ is the number of trials which are observed to have an outcome of true.

The conditional density $p_{T|\Theta}(t|\theta)$ itself serves as the characterization of the target referred to in previous sections. It provides the basis for the hypothesis testing needed to detect rare signature events in Section 4. Its dependence on certain geometric independent variables necessitates the tests of homogeneity in Section 5. An estimate $\hat{T}$ of a realization of T can be obtained as the maximum or mean of this conditional density.

Choosing this method of characterizing the probability of observing a stable target signature is a commitment to what is commonly called a Bayesian approach, which relies heavily on the modeling density $p_T(t)$. Other methods place more emphasis on the sampling density $p_{\Theta|T}(\theta|t)$. (The sampling density is required in the Bayesian approach also, but takes a more indirect role.) Therefore, in the sequel the important probability measure will be the conditional density $p_{T|\Theta}(t|\theta)$ which will be implied in the operations of expected value and variance. In order to reduce some of the notational burden the subscripts will be dropped from the probability density notations throughout the remainder of this section.

The density $p(t|\theta)$ can be written in terms of the densities $p(\theta|t)$ and $p(t)$ by writing the two equivalent expressions for the joint density of Θ and T as

$$p(\theta, t) = p(t|\theta)p(\theta) = p(\theta|t)p(t), \tag{3.1}$$

and noting that

$$p(\theta) = \int_0^1 p(\theta, t)\, dt = \int_0^1 p(\theta \,|\, t) p(t)\, dt. \tag{3.2}$$

It follows from Equations (3.1) and (3.2) that

$$p(t \,|\, \theta) = \frac{p(\theta \,|\, t) p(t)}{p(\theta)} = \frac{p(\theta \,|\, t) p(t)}{\int_0^1 p(\theta \,|\, t) p(t)\, dt}. \tag{3.3}$$

It follows from the assumptions that the density for the number of trials Θ which have an outcome of true out of N trials is given by a binomial density. Thus

$$p(\theta \,|\, t) = b(N, t, \theta) = \binom{N}{\theta} t^{\theta}(1-t)^{N-\theta}, \qquad 0 \le t \le 1. \tag{3.4}$$

It remains to choose a form for the density $p(t)$. A useful density for this purpose (called a beta density) is defined by

$$p(t) = p(t, \alpha, \beta) = \begin{cases} \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}, & 0 \le t \le 1, \quad \alpha \ge 0, \quad \beta \ge 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

where

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\, dx. \tag{3.6}$$

The function $B(\alpha, \beta)$ is called the beta function and is introduced to normalize the integral of Equation (3.5) to one. Restricting α and β to integer values results in the well-known identity

$$B(\alpha, \beta) = \frac{(\alpha-1)!(\beta-1)!}{(\alpha+\beta-1)!}, \qquad \alpha, \beta = 1, 2, 3, \ldots \tag{3.7}$$

which can be obtained by repeated application of a standard integral reduction formula.

Combining Equations (3.4) and (3.5), the numerator of Equation (3.3) is

$$p(\theta \,|\, t) p(t) = \binom{N}{\theta} \frac{t^{\theta+\alpha-1}(1-t)^{N-\theta+\beta-1}}{B(\alpha, \beta)}, \qquad 0 \le t \le 1, \quad \alpha \ge 0, \quad \beta \ge 0. \tag{3.8}$$

Integrating Equation (3.8) determines the denominator of Equation (3.3) to be

$$\int_0^1 p(\theta \,|\, t) p(t)\, dt = \binom{N}{\theta} \frac{B(\theta+\alpha, N-\theta+\beta)}{B(\alpha, \beta)}. \tag{3.9}$$

from the definition of the beta function [Equation (3.6)].

Finally, by combining Equations (3.8) and (3.9), it can be seen that the conditional density [Equation (3.3)] is another beta density

$$
\begin{aligned}
p(t\,|\,\theta) &= \begin{cases} \dfrac{t^{\theta+\alpha-1}(1-t)^{N-\theta+\beta-1}}{B(\theta+\alpha, N-\theta+\beta)}, & 0 \le t \le 1, \quad \alpha \ge 0, \quad \beta \ge 0 \\ 0, & \text{otherwise} \end{cases} \\
&= p(t, \theta+\alpha, N-\theta+\beta). \qquad (3.10)
\end{aligned}
$$

A comparison of the a priori probability density [Equation (3.5)] with the conditional density [Equation (3.10)] shows the effect of observing an outcome of N random trials on the density parameters.

Typical numerical results are illustrated in Figure 3-1 where densities for two different targets have been computed based on twenty signatures from each target.

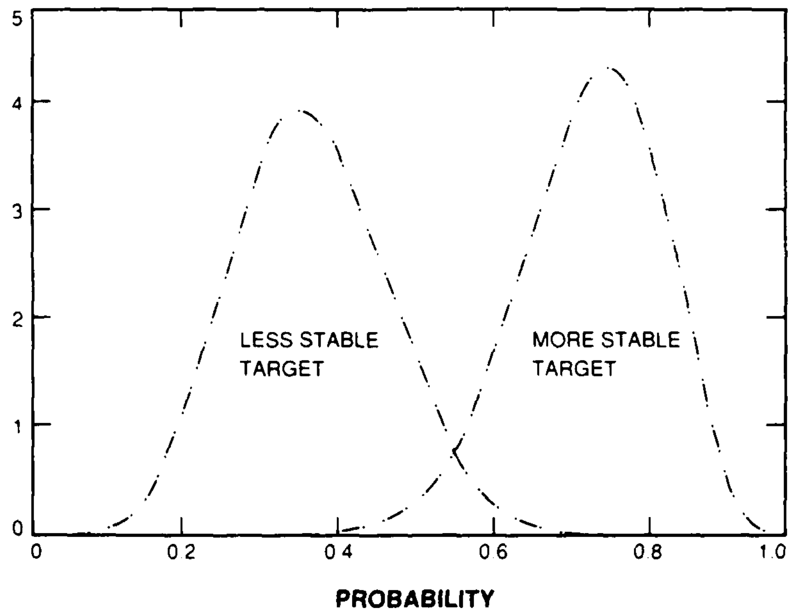

Figure 3-1. These densities were computed from Equation (3.10) using twenty signatures from each of two targets. Typically, the computed density characterization of a target whose signatures usually appear unstable has most of its probability mass to the left of the density characterization of a target whose signatures usually appear stable.

In both cases a uniform prior density $p(t)$ was assumed by setting $\alpha = \beta = 1$. The difference in the computed densities reflects the fact that one target, which is inert, is less likely to appear stable than the other target, which is under active attitude control.

The mean $\mathbf{E}[T\,|\,\theta]$ of the conditional density [Equation (3.10)] is given by

$$
\begin{aligned}
\mathbf{E}[T\,|\,\theta] &= \frac{1}{B(\theta+\alpha, N-\theta+\beta)} \int_0^1 t \cdot t^{\theta+\alpha-1}(1-t)^{N-\theta+\beta-1}\,dt \\
&= \frac{B(\theta+\alpha+1, N-\theta+\beta)}{B(\theta+\alpha, N-\theta+\beta)} \\
&= \frac{(\theta+\alpha)!}{(N+\alpha+\beta)!} \cdot \frac{(N+\alpha+\beta-1)!}{(\theta+\alpha-1)!} \\
&= \frac{\theta+\alpha}{N+\alpha+\beta}, \qquad \alpha, \beta = 1, 2, 3, \ldots
\end{aligned} \tag{3.11}
$$

where α and β are restricted to integer values because the special case evaluation of the beta function [Equation (3.7)] has been used.

Similarly compute

$$
\begin{aligned}
\mathbf{E}[T^2\,|\,\theta] &= \frac{1}{B(\theta+\alpha, N-\theta+\beta)} \int_0^1 t^2 \cdot t^{\theta+\alpha-1}(1-t)^{N-\theta+\beta-1}\,dt \\
&= \frac{B(\theta+\alpha+2, N-\theta+\beta)}{B(\theta+\alpha, N-\theta+\beta)} \\
&= \frac{(\theta+\alpha+1)!}{(N+\alpha+\beta+1)!} \cdot \frac{(N+\alpha+\beta-1)!}{(\theta+\alpha-1)!} \\
&= \frac{(\theta+\alpha+1)}{(N+\alpha+\beta+1)}\mathbf{E}[T\,|\,\theta], \qquad \alpha, \beta = 1, 2, 3, \ldots
\end{aligned} \tag{3.12}
$$

which is written in terms of the conditional mean, Equation (3.11).

If the a priori density $p(t)$ for T is given by Equation (3.5), then after N trials of which Θ had an outcome of true, an estimate $\hat{T}$ of the realization of T is given by

$$
\hat{T} = \mathbf{E}[T\,|\,\Theta] \tag{3.13}
$$

and its mean squared error over the ensemble of experiments consisting of N trials, Θ of which have outcomes of true is

$$
\begin{aligned}
\mathbf{E}[(T-\hat{T})^2\,|\,\Theta] &= \mathbf{E}[T^2\,|\,\Theta] - \hat{T}^2 \\
&= \left(\frac{\Theta+\alpha+1}{N+\alpha+\beta+1} - \hat{T}\right)\hat{T}, \qquad \alpha, \beta = 1, 2, 3, \ldots.
\end{aligned} \tag{3.14}
$$

The estimate [Equation (3.13)] provides some indication of the probability of observing a stable signature for a target based on its characteristics over the previous N tracks. By providing a measure of estimate error, the mean squared error [Equation (3.14)] can be used to determine if N tracks is enough to estimate the probability with the desired accuracy.

Necessary conditions of optimality, which set the derivative of Equation (3.14) with respect to Θ to zero, indicate that an extreme value of Equation (3.14) occurs at $\Theta = (N + \beta - \alpha)/2$. This turns out to be its maximum. Therefore an upper bound on the mean squared error of Equation (3.14) is given by

$$\mathbf{E}[(T - \hat{T})^2 \mid \Theta] \leq \mathbf{E}[(T - \hat{T})^2 \mid \Theta = (N + \beta - \alpha)/2] = \frac{1}{4(N + \alpha + \beta + 1)} \tag{3.15}$$

For the case that $\alpha = \beta = 1$ the prior density $p(t)$ is uniform and the square-root of Equation (3.15) is plotted versus N in Figure 3-2.

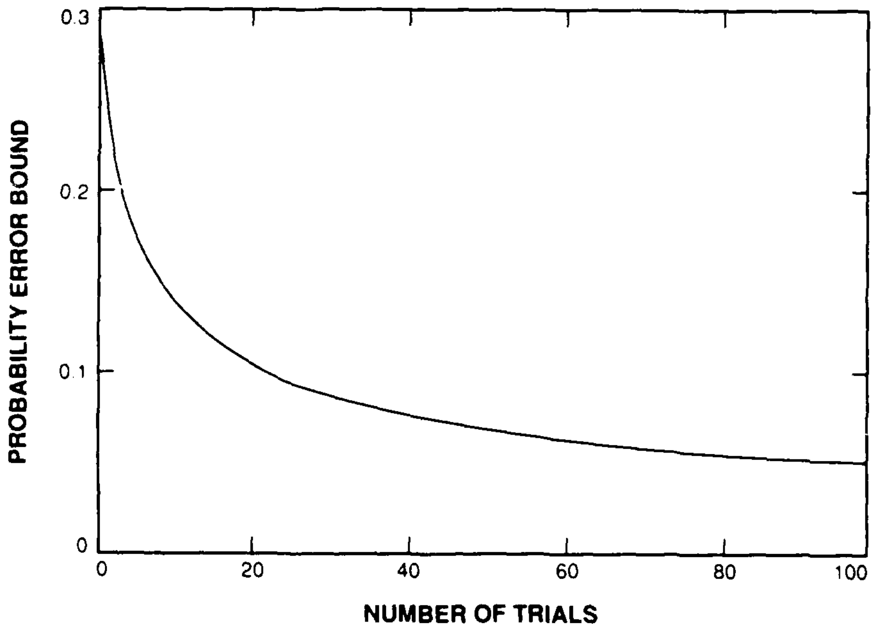

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Figure 3-2. An upper bound on the square-root of the mean squared error [Equation (3.14)] of the estimate [Equation (3.13)] of the probability of observing a stable signature is plotted versus the number of signatures N already observed under the assumption of a uniform prior density for $p(t)$.

The estimate of Equation (3.13) and its error [Equation (3.14) or (3.15)] are useful in assessing whether an observed trial, either a stable or unstable signature, is a statistically rare behavior of the target. Procedures for uncovering such rare behaviors are made more precise in Section 4 where the conditional density [Equation (3.10)] is used to directly test for evidence of unusual, or changing, stability behavior of the target. As seen from Figure 3-2, the error in the point estimate of probability might be fairly large, greater than 0.1, until more than 20 trials have been conducted. Fewer trials are needed if the only interest concerns whether or not T lies within a range of values.

This application, an interval estimate, will turn out to be the most important and is developed next.

4. RARE EVENT DETECTION

The first application of the probability density characterization of the target will be to automatically isolate examples of rare behavior of the target signature. The behavior in this case is the apparent stability, or lack of it, as evidenced by examples of target signatures. Rare behavior observed in a new target signature might indicate a change in target behavior or otherwise provide an indication of its operational characteristics.

The probability density [Equation (3.10)] incorporates the previously observed target signatures into a characterization of the probability of observing a stable or unstable target signature. It is used to formulate the appropriate test to determine if stability or instability is a rare behavior of the target, and then isolate any new signatures (either the stable or the unstable) which represent the distinct minority. This is easily done because of the work accomplished in the previous sections.

Precisely, a *rare (signature) event* is either a true or a false outcome of a trial (the result of testing a signature) that occurs with probability less than $\mathcal{R}$. That is, a rare event is an experimental outcome of true when T (the probability of observing an outcome of true) is less than $\mathcal{R}$, or an experimental outcome of false when $F = 1 - T$ (the probability of observing an outcome of false) is less than $\mathcal{R}$. To be physically intuitive, the value of $\mathcal{R}$ should be small, certainly less than one half.

Similarly, a *common event* is an outcome of a trial that occurs with probability greater than $1 - \mathcal{R}$. This definition is convenient for discussion, but redundant because it follows from

$$T < \mathcal{R} \iff F > 1 - \mathcal{R}, \qquad \text{and} \qquad F < \mathcal{R} \iff T > 1 - \mathcal{R} \tag{4.1}$$

that an outcome of true is rare whenever an outcome of false is common, and an outcome of false is rare whenever an outcome of true is common.

Although an observed signature is either rare or not, the observer does not know a priori which is the case because specific knowledge of the value of T is lacking, and there is always some uncertainty in deciding if a signature should be declared rare based on an interpretation of previous trials. The first encounter of this sort of uncertainty was in Section 2 where a two-tailed test was proposed to decide if the signature itself was stable or unstable—there was a (controlled) probability that an incorrect decision might be made under an assumption of stability.

At some point an observer is asked to await the next trial and either declare it to be rare or not. At this time, N previous experiments (where N might be 0) have been performed. It is recognized from Equation (3.10) that the probability P_f that an experimental outcome of false is rare is estimated by

$$P_f = \int_{1-\mathcal{R}}^{1} p(t, \Theta + \alpha, N - \Theta + \beta)\, dt. \tag{4.2}$$

Similarly, the probability P_t that an experimental outcome of true is rare is estimated by

$$P_t = \int_0^R p(t, \Theta + \alpha, N - \Theta + \beta)\, dt. \tag{4.3}$$

Because each trial either represents a rare event or not, the probability that an outcome of true is not rare must be $1 - P_t$. Similarly, the probability that an outcome of false is not rare must be $1 - P_f$. A probability threshold ρ can be chosen, suppose on the probability of error. Then a trial with an outcome of true is declared a rare event whenever $P_t > 1 - \rho$. Similarly, a trial with an outcome of false is declared a rare event whenever $P_f > 1 - \rho$.

An outcome of a trial need not represent either a rare or a common event; perhaps not enough signatures have been tested for the probability threshold $1 - \rho$ to be crossed. To allow for this possibility the notion of an armed introduced. If it is possible, depending on the trial outcome, to declare a rare event on the next trial, the test for rare events is said to be armed.

When a test is armed the hypothesis that the probability T is either greater than $1 - \mathcal{R}$ or less than $\mathcal{R}$ has been accepted with probability at least $1 - \rho$, and the next experimental outcome can be declared either rare or common. When the test is armed for a particular target, its signatures are actually being monitored for unusual behavior. If the test is not armed for a particular target, then the target is not currently being monitored for unusual behavior.

Figure 4-1 graphically presents some the results concerning rare event detection for a target which is generally stable. The graph plots several probabilities versus time for the target. A point probability estimate based on Equation (3.13) is plotted along with the probability that T is greater than $1 - \mathcal{R}$ with $\mathcal{R} = 0.2$. Suppose that the probability threshold for declaring a rare event is 0.95. Then the latter probability crosses this threshold 18 days (and 14 tracks) after the system is *initialized, arming the test*. Stable behavior is declared common for this target, and an unstable signature will be declared rare. No rare (unstable) events occur however for another 12 days, when the first rare unstable signature is observed. The second rare unstable signature is observed 10 days later. (Actually, on that day 2 consecutive unstable signature events were observed within a few minutes of each other.)

A typical stable signature for this object is shown in Figure 4-2 and the two rare unstable signatures are shown in Figure 4-3.

Two signatures out of 132 were identified by the test for rare events, greatly reducing the number of signatures which need to be manually reviewed to monitor the behavior of the target. These unstable signatures are typical of small changes in target attitude, not a loss of attitude control; but had a catastrophic destabilization of the target occurred after day 18 of monitoring, it would have been immediately detected by the test.

The final application of the density characterization of the target involves studying its dependence on various independent variables. The density may change, subject to where in the orbit the target signature is collected. This is because the normal behavior of the target may depend on orbital variables. Also, the density might change depending on the sensor viewing geometries. Testing the dependency of the probability density [Equation (3.10)] on these variables is accomplished by invoking the tests of homogeneity in Section 5.

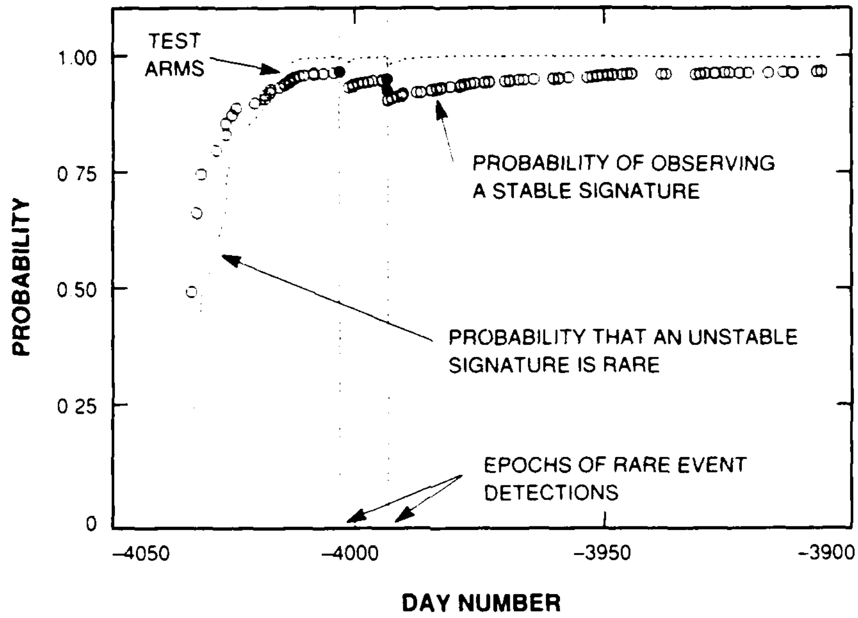

Figure 4-1. Some of the probabilities computed in the rare event detection problem are shown for a target which usually appears stable. Three rare events in 132 signatures were automatically detected. Hollow symbols represent signatures declared to be stable while filled symbols represent signatures declared to be unstable.

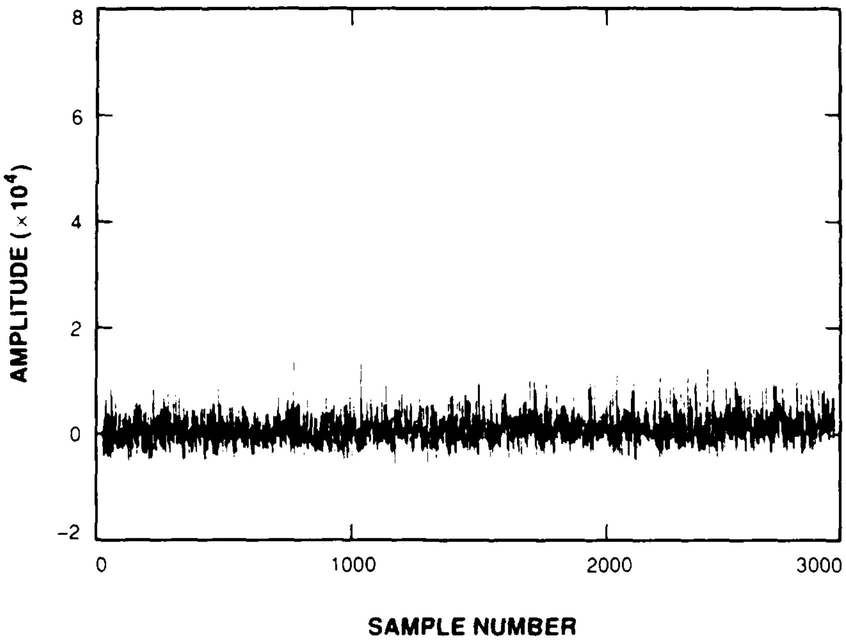

Figure 4-2. The common stable signatures of the target all appear similar to this one.

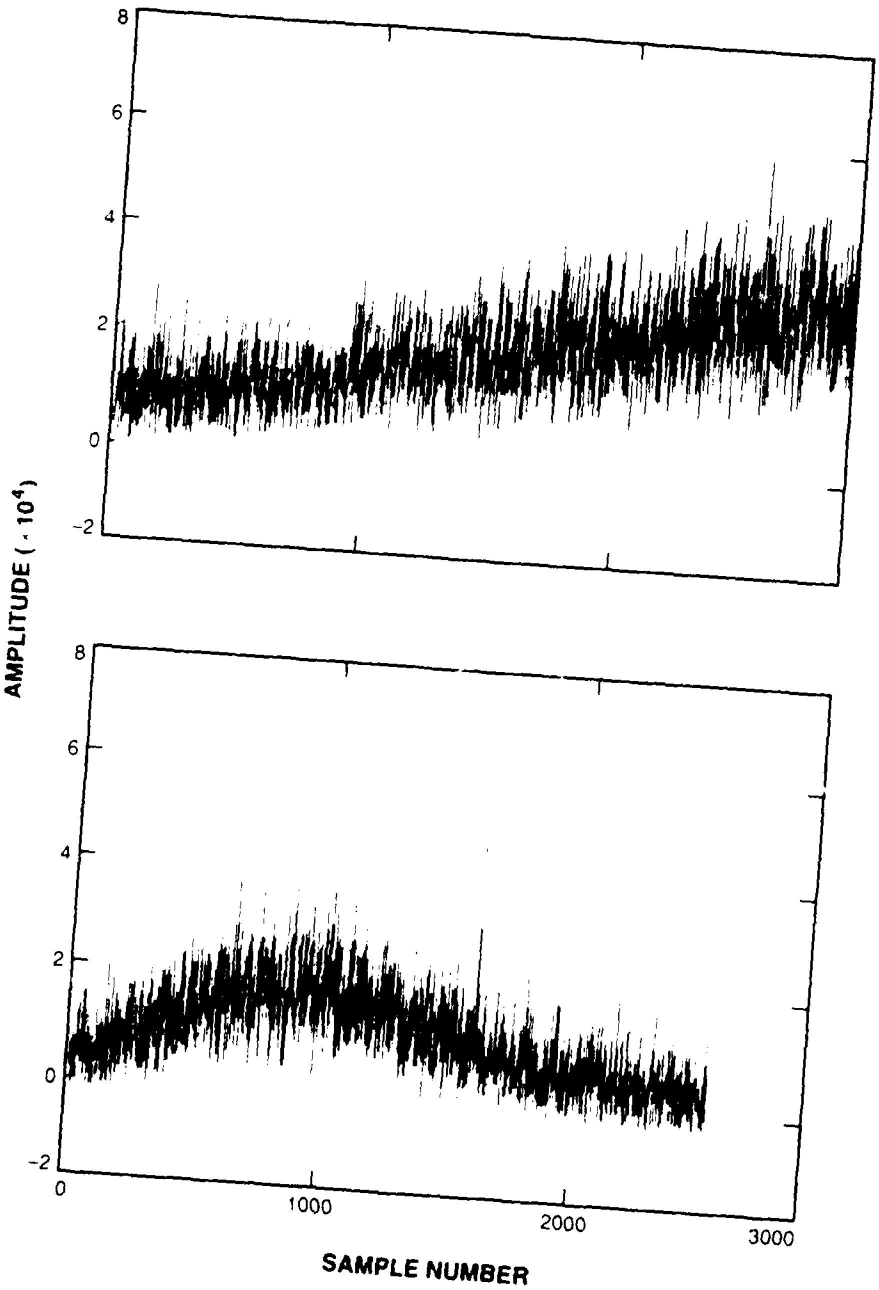

Figure 4-3. Three rar signatures for the target show a trend and a lobe-like structure. These are typical of a small change in target attitude, not a catastrophic loss of stability.

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5. HOMOGENEITY OF THE CLASSIFICATION STATISTICS

Up to this point the signature runs statistics for a given target have been treated as a time series, i.e., a discrete real (random) function of time. It may be the case that other independent variables provide a more natural coordinate system for characterizing target behavior. For example, if the target consistently points at the center of the Earth, then sensor elevation, and not time, may be a more natural independent variable. Some targets in eccentric orbits might perform certain functions in relationship to apogee or perigee so that the true anomaly may be an important variable. Solar cell panels may be kept oriented with respect to the sun making the angle between the sensor-to-sun vector and the sensor-to-target vector important (called the sun-sensor-target angle) and so forth.

In Figure 5-1, the runs statistics from many signatures of a target are plotted against time and the sun-sensor-target angle. The thresholds for the two-tailed test accepting the stability hypothesis are shown as dotted lines for a significance of 10^{-3}. From the plots it is not unreasonable to conclude that the stability hypothesis is rejected randomly over time, but that rejection is highly dependent on the sun-sensor-target angle. This suspicion will be verified by the tests of homogeneity developed in this section.

From now on other well orderings of the target signature runs statistics are considered. The index of the statistics $\{X_i\}$ might correspond to increasing elevation, for example, rather than increasing time. In particular, the dependence of target probability density characterization (as developed in Section 3) on these geometric variables is considered. A simple random process model for the sequence of target signature runs statistics is proposed which consists of an ordered sample of independent random variables X_i written as

$$X = \{X_1, X_2, \ldots X_{L+M}\} = \{Y_1, Y_2, \ldots Y_L, Z_1, Z_2, \ldots Z_M\}$$

where

$$Y_i \sim f_Y(y),\ Z_i \sim f_Z(z) \text{ and } f_Y(y) \neq f_Z(z). \tag{5.1}$$

Thus the postulated dependence of the probability density characterization of the target on the independent variable is a simple one—samples from one of two distinct populations are observed, depending on whether or not the value of the independent variable exceeds a threshold. It is reasonable to attempt to determine a confidence interval for L, often called the disorder value of the process [6], from a realization of X. Such nonstationarity of signature process models has been demonstrated before (for example, in [7]) where detrended signature data have been successfully fit to autoregressive process models within a track in order to detect time transitions within the signatures. These transitions are loosely analogous to the above disorder values. Some differences between that application and the one in this report include the use of independent variables other than time and different random process models.

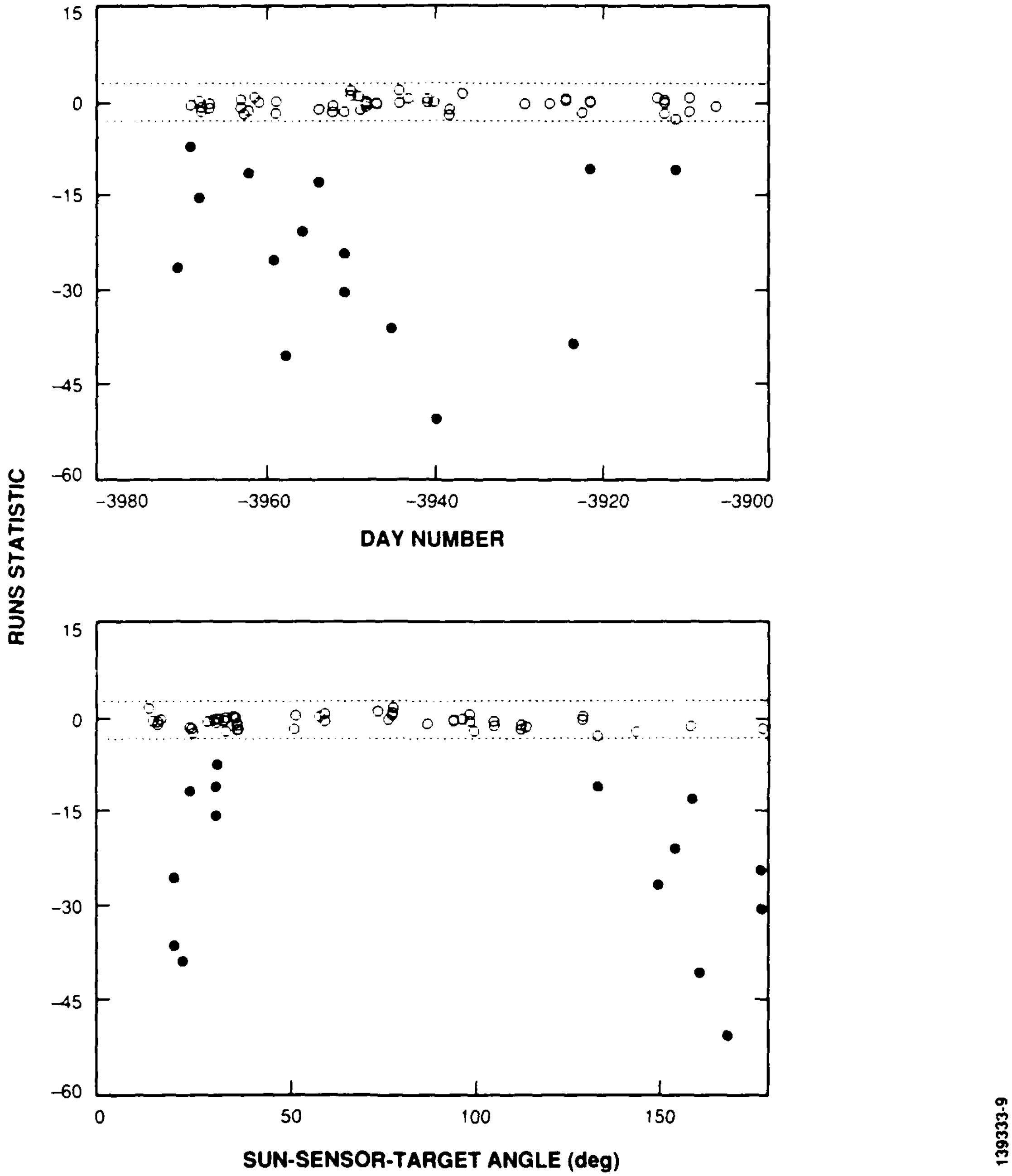

Figure 5-1. The runs statistics for a target are considered as functions of time and the sun-sensor-target angle. The plots suggest that the failures of the test for stability are dependent on the sun-sensor-target angle, but occur randomly in time. Hollow symbols represent signatures declared to be stable while filled symbols represent signatures declared to be unstable.

There are several ways to pose an appropriate question about the sequence of target signature runs statistics. Assuming that $X_i \sim f_{X_i}(x_i)$, the general question of how the densities $f_{X_i}(x_i)$ depend on i is the problem of homogeneity in statistics. Again, nonparametric statistics provides several useful approaches. Equation (5.1) suggests that a two-sample method (using a rank statistic) such as originally proposed by Mann and Whitney [8] or Wilcoxin [9] could be used. A more recent discussion of these methods can be found in [10]. Other possibilities include the two-sample adaptation of the Kolmogorov-Smirnov test [5] and a method of Wald and Wolfowitz described in [11].

One interesting way to use these tests is to extremize the test statistic as a function of a point estimate $\hat{L}$ of L. In [12], for example, Darkhovskh shows that such a parameterized test can be used as a consistent estimator of the disorder value L. The problem with this approach is that no measure of the error in the estimate is provided.

The following alternate approach based on hypothesis testing is proposed. This approach yields an interval containing L with some probability, not a point estimate $\hat{L}$ of L. Note that the model of Equation (5.1) satisfies the following:

1. L runs statistics indexed from 1 to L have the same distribution.
2. M runs statistics indexed from $L+1$ to $L+M$ have the same distribution, and
3. Runs statistics indexed from 1 to L do not have the same distribution as the runs statistics indexed from $L+1$ to $L+M$.

The approach involves capturing these features in a set of hypotheses that can be tested to develop a confidence region for the disorder value L. Assuming that $X_i \sim f_{X_i}(x_i)$ for $1 \le k \le L+M$, define the hypotheses

$$H_Y(k): \qquad f_{X_1}(x_1) = f_{X_2}(x_2) = \ldots f_{X_k}(x_k),$$

$$H_Z(k): \qquad f_{X_{k+1}}(x_{k+1}) = f_{X_{k+2}}(x_{k+2}) = \ldots f_{X_{L+M}}(x_{L+M}),$$

and

$$H_B(k): \qquad f_{X_l}(x_l) = f_{X_m}(x_m) \text{ for } 1 \le l \le k \text{ and } k < m \le L+M. \tag{5.2}$$

Rather than introduce new distribution theory, the theory of runs summarized in Section 2 will be used to compute a test statistic and its distribution under each of the hypotheses in Equation (5.2). The method discussed in Section 2 can be used directly to test hypotheses $H_Y(k)$ and $H_Z(k)$. The Wald-Wolfowitz method [11], [5] can be used to test $H_B(k)$.

An implementation of the Wald-Wolfowitz test begins with the following transformation. Form the sequence $W = \{W_1, W_2, \ldots W_{L+M}\}$ of the geometric variables of interest such that W_i is the value of the independent variable associated with X_i, e.g., W_i was the observed value of the independent variable when the runs statistic X_i was observed. By construction, $\tilde{W}_{i+1} \geq \tilde{W}_i$ for $i = 1, 2, \ldots L + M - 1$. By a reordering is meant an invertible function $r : I \rightarrow I$ from $I = \{1, 2, \ldots L + M\}$ onto I. Define the sequence

$$W' = \{W'_1, W'_2, \ldots W'_{L+M}\} \tag{5.3}$$

by setting

$$W'_i = \tilde{W}_{r(i)} \qquad \text{for } i = 1, 2, \ldots L + M \tag{5.4}$$

where the reordering r satisfies

$$m \geq n \Longrightarrow X_{r(m)} \geq X_{r(n)}. \tag{5.5}$$

The result is that the sequence of geometric variable values W' have been reordered to correspond with increasing values of the associated signature runs statistics. To test $H_B(k)$ based on a method of Wald and Wolfowitz [11], the runs test of Section 2 can be applied to W' with the threshold set to $(\tilde{W}_k + \tilde{W}_{k+1})/2$.

Thus it is possible to compute the sets

$$S_Y = \{i \mid H_Y(i) \text{ is accepted with significance } \alpha_Y\}, \tag{5.6}$$

$$S_Z = \{i \mid H_Z(i) \text{ is accepted with significance } \alpha_Z\}, \tag{5.7}$$

and

$$S_B = \{i \mid H_B(i) \text{ is accepted with significance } \alpha_B\}. \tag{5.8}$$

Recall that the significance of a test is the probability that the test will reject the null hypothesis when it is true, so that generally the significance is set to a small value. A confidence region C_L for the disorder value L is given by

$$C_L = S_Y \cap S_Z \cap S_B^c, \tag{5.9}$$

where S_B^c denotes the complement of S_B.

The probability that the disorder value L will be erroneously omitted from C_L cannot be computed without a specific assumption on the distribution of the runs statistics under a hypothesis corresponding to the alternate of H_B. However, a lower bound on this probability is obtained by realizing that such an error will be made at least as often as either $H_Y(L)$ or $H_Z(L)$ are erroneously rejected. Thus

$$\begin{aligned} P[L \notin C_L \mid L \text{ is a disorder value}] &\geq \alpha_Y \alpha_Z + \alpha_Y(1 - \alpha_Z) + \alpha_Z(1 - \alpha_Y) \\ &= \alpha_Y - \alpha_Y \alpha_Z + \alpha_Z. \end{aligned} \tag{5.10}$$

The bound will tighten as the difference between $f_Y(y)$ and $f_Z(z)$, as measured by the test of $H_B(L)$, becomes greater.

Under the hypothesis that $f_Y(y) = f_Z(z)$, a bound on the probability that a value k will be erroneously placed in C_L is obtained by realizing that such an error will occur no more often than $H_B(k)$ is erroneously rejected. Thus

$$P[k \in C_L \mid f_Y(y) = f_Z(z)] \leq \alpha_B. \tag{5.11}$$

This provides some certainty that a disorder value will not be declared when none exists, but good behavior of the confidence interval under other alternative hypotheses cannot be guaranteed.

Now define the sets

$$X' = \{X_i \in X \mid i < k \text{ for all } k \in C_L\} \tag{5.12}$$

and

$$X'' = \{X_i \in X \mid i > k \text{ for all } k \in C_L\}. \tag{5.13}$$

These sets can be used to replace the original set X of runs statistics by two smaller sets with some confidence that the two density characterizations obtained from the smaller sets via the techniques of Section 3 will be significantly different.

In Figure 5-2 some of the probabilities associated with rare event detection are plotted for the target whose runs statistics were used in Figure 5-1. This is analogous to Figure 4-1 except that the test of homogeneity has first been applied with 10^{-2} significance to replace the original sequence of runs statistics with the two smaller sets given by Equations (5.12) and (5.13)—one containing the trials conducted within 35° of solar alignment, and the other containing trials conducted greater than 50° from solar alignment.[1] Here solar alignment is the condition that the sun-sensor-target angle is either 0° or 180°. It can be seen from the point estimate of the probability of observing a stable signature in the two regions (and by recalling the example of Figure 3-1) that the densities themselves must be quite different. Stability is a common event away from solar alignment—the test is armed in this region. Neither stability nor instability are rare near solar alignment—the test is not armed in this region. No rare events have been detected.

If the test of homogeneity had not been applied then the evolution of probabilities in the rare event detection problem appear as in Figure 5-3. The test for rare events is not armed in this case and the target is not being monitored for rare signature events in any portion of the orbit. If the test of homogeneity had not been used and the test for rare events had armed, then many rare signature events might have been declared. Therefore another important effect of the test for

[1] Due to the scope of the project, only a partial implementation of the test described in this section was implemented. Only the set S_B^c was computed in expression (5.9), yielding an upper bound on the interval C_L.

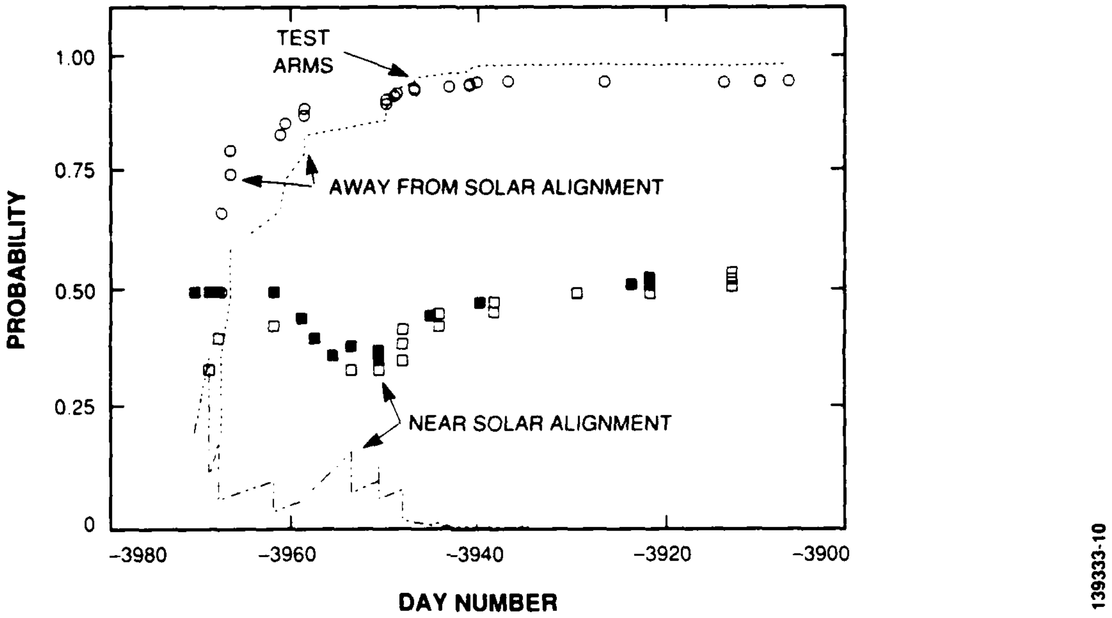

Figure 5-2. The test of homogeneity has established the dependence of the probability of observing a stable target signature on the sun-sensor-target angle for this target.

homogeneity in some cases is to reduce the number of rare events by disarming the test in the region of the independent variable where globally rare behavior is locally not rare.

This behavior is illustrated by the examples in Figure 5-4. Probabilities for another target are shown with and without the test of homogeneity. Without the test for homogeneity the dependence of the probabilities on the sun-sensor-target angle is ignored, the test for rare events arms and seven rare events are detected. (Two of them occur nearly simultaneously and are hard to distinguish in the top graph.) These seven rare signature events, which are all observations of an unstable signature, occur near solar alignment. After the test for homogeneity has been applied with 10^{-2} significance, the signatures are partitioned into two sets corresponding to Equations (5.12) and (5.13). Each signature from the set taken near solar alignment occurs within 15° of alignment—this is the only portion of the orbit where the unstable signatures are observed. The other set of signatures all occur greater than 25° from solar alignment.

After application of the test of homogeneity the test for rare events does not arm near solar alignment, hence no rare events are declared. The results of the test of homogeneity suggest that stability for this target is common away from solar alignment, as it does for the target in Figure 5-2. In this case the test for homogeneity has also reduced the number of rare event detections, by correctly associating unstable signatures with a region of the orbit where they are not rare.

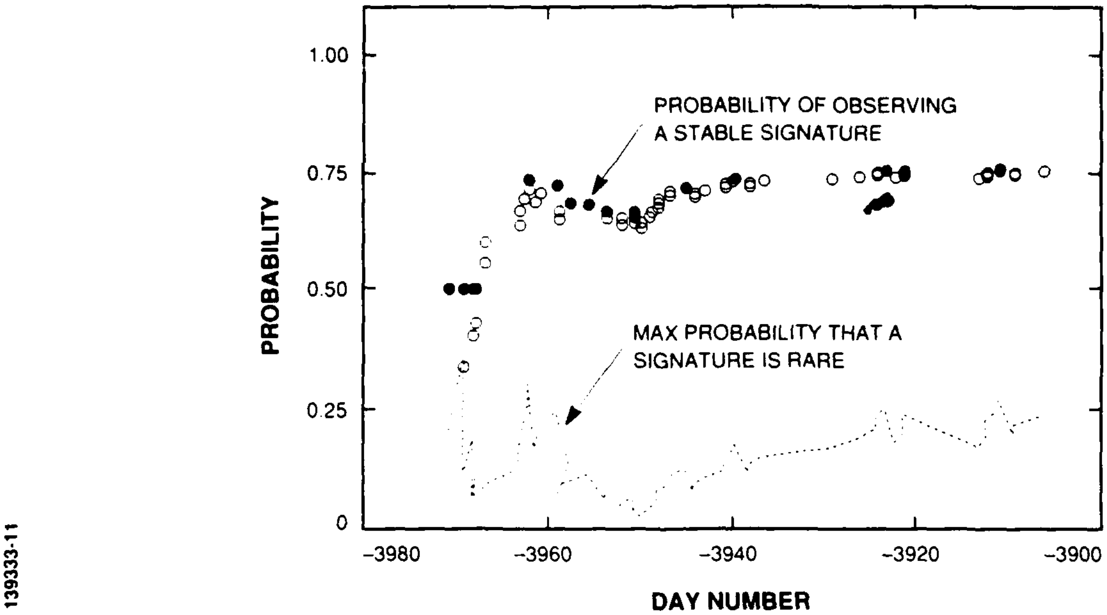

Figure 5-3. Without the test of homogeneity the test for rare events does not arm (no rare events can be declared) and the dependence of the probability of observing a stable target signature on the sun-sensor-target angle is not obvious.

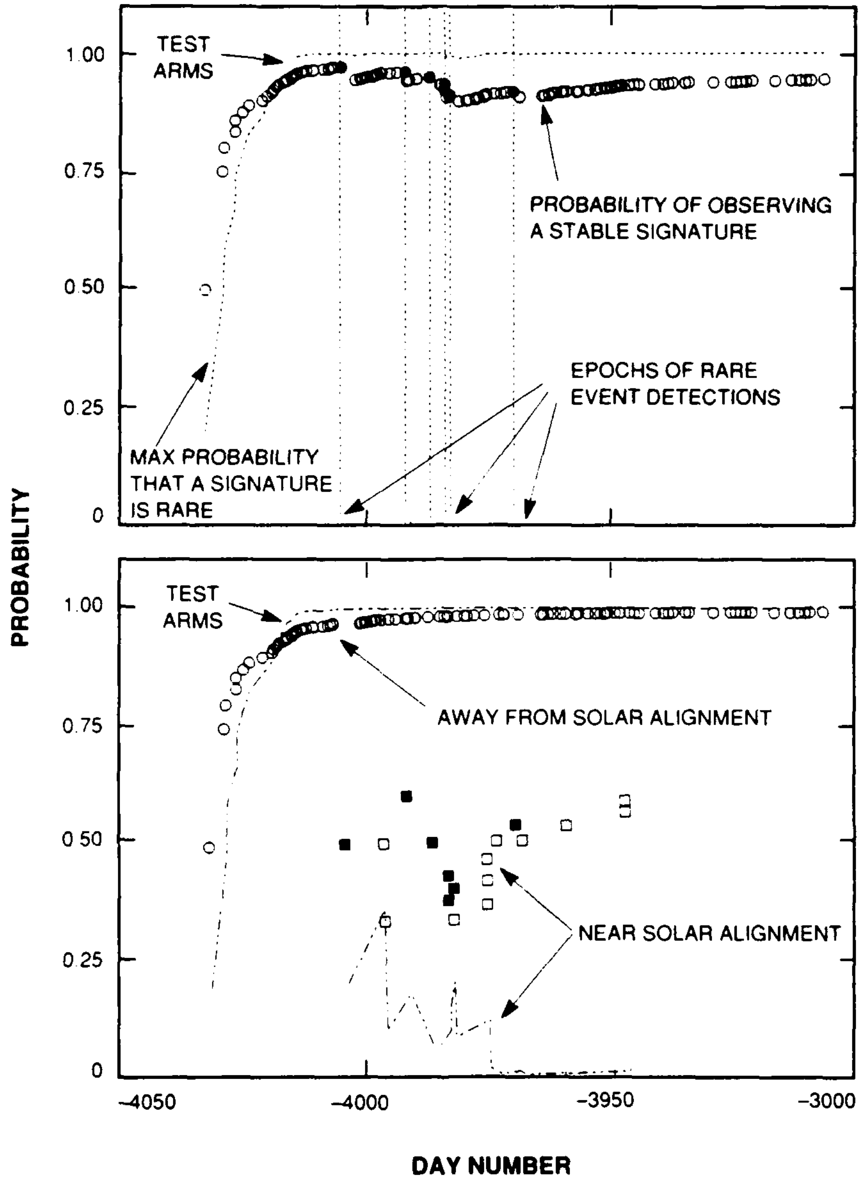

Figure 5-4. Besides establishing the dependence of the probability of observing a stable target signature on the sun-sensor-target angle for a given target, the test of homogeneity can reduce the number of rare event detections by correctly associating stability behavior with the appropriate portion of the orbit.

6. SUMMARY

Based primarily on issues of availability, approximately 1600 signatures from 38 high-altitude targets were collected for processing with algorithmic implementations of the theory presented in this report. The number of signatures per target ranged from 17 to 158, with most targets yielding less than 50. The data were collected over approximately an 80-day span (see Table 6-1). The examples used to illustrate the ideas in this report were mostly drawn from this population. Throughout the tests, levels of significance were fixed and not varied on a target-by-target or signature-by-signature basis. Varying the parameters might undesirably skew these results with respect to the performance obtained in an automated system. All the results depend, however, on the population of targets selected. It is felt that these results might be typical of high-Earth-orbit targets sensed with intermediate wavelength radar.

TABLE 6-1.

Summary of Experimental Results

	Targets	Percent of All Targets	Tracks	Percent of All Tracks	Rare Event Detections	Percent of Tracks
Totals	38	100	1607	100	32	2.0
Armed Tests	30	79	1357	84	32	2.3
Solar Dependent	10	26	629	39	8	1.3

The flow of data and the computational steps are illustrated in Figure 6-1. There are many generalizations from the two-Bernoulli-trial case to M-Bernoulli trials, but none are shown in the figure because only a two-trial model [Equation (5.1)] was used in this report. Each of the computations shown in the figure have been previously described in this report and each computation was applied to every target and signature obtained for the numerical work.

A 10^{-3} significance was used in the signature classification test of Section 2. Because all the signatures consisted of between 800 and 10000 samples, the large population statistic [Equation (2.3)] was used and tested against the normal distribution. The notion of rare was quantified to mean "occurring less than 20% of the time" so that $\mathcal{R}$ was set to 0.2. At the conclusion of the experiment the test for rare events had armed for 30 of the 38 targets—armed being the condition indicating that the target has been successfully characterized and rare events can be declared (See a more detailed discussion in Section 4.) The notion of an armed test is necessary because a practical algorithm cannot be allowed to draw conclusions from insufficient data. All rare events were declared with a significance of 0.05. Almost 1400 of the signatures collected were from targets for which the test had armed.

The test for rare signature events detected 32, about 2% of the total number of signatures processed and less than 3% of the signatures processed through armed tests. Then for this population at least, 98% of the signatures were handled by the algorithm—the remaining 2% might be given to an analyst who could attempt to determine the importance of the rare signature events. For all of the targets used in the experiment, stability was common and instability was rare.

One of the first things that an analyst might have noticed is that in some cases, although the rare events seemed to occur randomly in time, they were dependent on other geometric variables. The idea of quantifying this dependence was formalized in the tests of homogeneity of Section 5, which were run against the data for the sun-sensor-target variable.[1] Only this one independent variable was tested. With a 0.05 significance, 10 of the 38 objects showed statistically significant dependence on the sun-sensor-target angle.

[1] Due to the scope of the project, only a partial implementation of the test described in Section 5 was implemented. Only the set S_B^c was computed in expression (5.9), yielding an upper bound on the interval C_L.

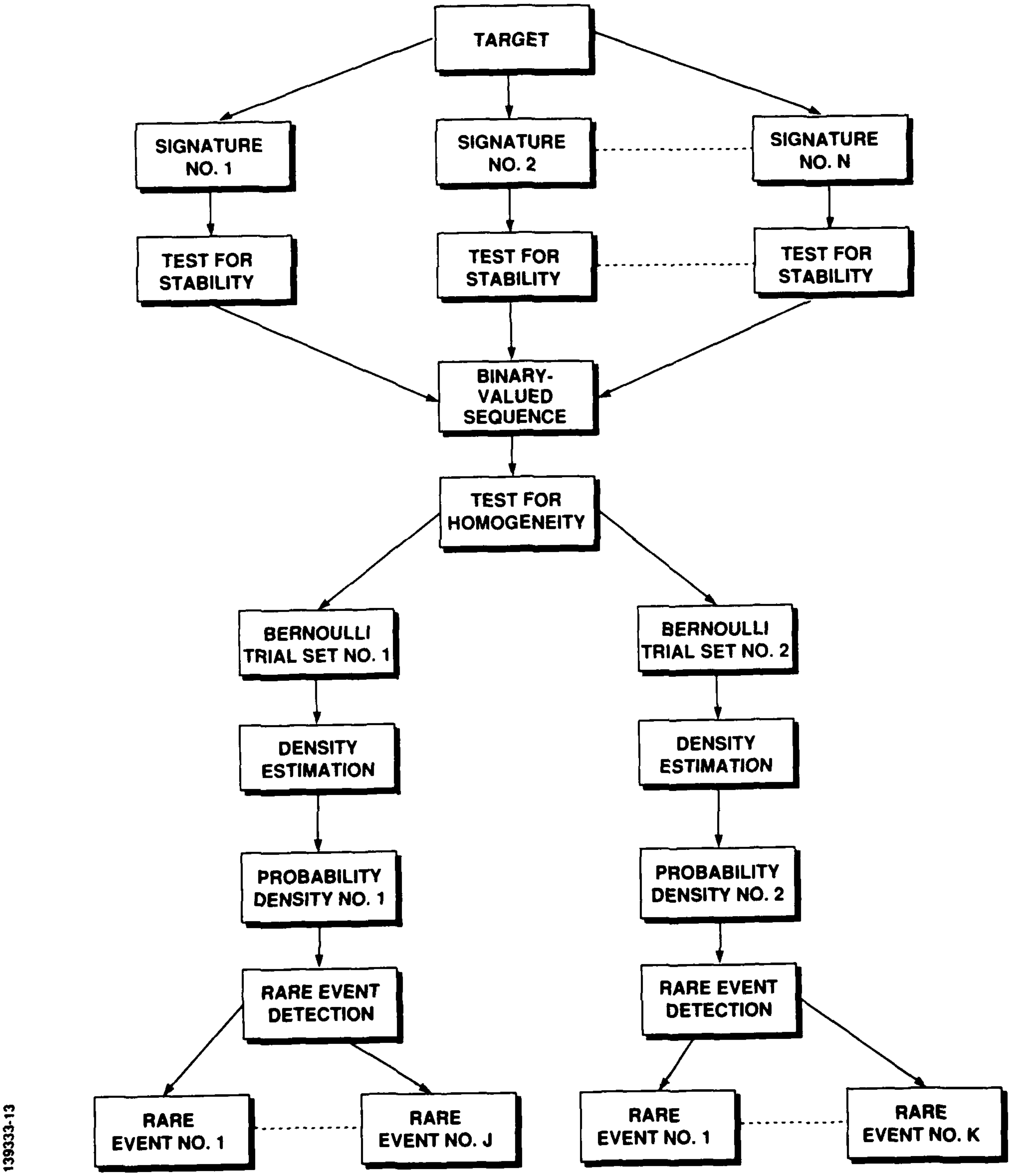

Figure 6-1. The computations and data flow associated with Table 6-1.

7. DISCUSSION

Robust and simple methods are needed to process the large amount of signature data produced during the routine ground-based tracking of a catalog of orbiting targets. Currently, most of these data are discarded. Calibrated cross-section time series are the most sophisticated techniques which could be currently used to represent such a large and diverse set of measurements. But cross-section time series fail to fuse, or relate, information from multiple tracks to form a characterization of the target behavior. Furthermore, there are too many signatures available to examine them all manually.

The theory behind a single bit processing system was described, using a variety of classical methods from nonparametric statistics, estimation, and hypothesis testing theories. A series of signatures obtained from the target over time are transformed into a time series consisting of only a single bit of information from each signature. The binary-valued series captures a description of the apparent target attitude stability, or lack of it, as a function of time and orbital geometry. Under a reasonable set of physical circumstances this series can be treated as a set of Bernoulli trials, and the associated probability density can be estimated. The density becomes a characterization of the target over its orbit and perhaps its operational life.

The density can be used in at least two ways as a characterization of the target behavior. First, by characterizing the normal behavior of the target, the algorithmic detection of rare signature events becomes possible. If a target normally appears stable, signatures characterizing an instability are flagged. If a target normally appears unstable, signatures characterizing stability are flagged. For some targets neither behavior is rare, and no rare signature events are *detectable*. *Second*, by examining the dependence of the probability density on the value of independent geometric variables, such as true anomaly or the angle between the sensor-to-sun vector and the sensor-to-target vector, attitude behavior related to the orbital position of the target becomes apparent.

Using these techniques, new information fused from a series of tracks becomes available to analysts. It would be difficult, at best, to intuit these results by visual inspection of the large number of original signatures. The results are free in the sense that the techniques can be applied to signatures obtained as a by-product of tracking targets for positional measurements. Only a small fraction of the original number of signatures, the ones declared to be rare, need to be analyzed manually—thus a useful bulk filtering function is performed and the occurrence of the rare event might be evidence of a change in the behavior of the target.

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A method of imposing a binary classification on a target signature is described that transforms a sequence of signatures into a binary-valued time series using the theory of runs. Under the assumption that the time series can be treated as a set of Bernoulli trials, a Bayesian method of estimating a probability density characterizing the outcome of each trial is considered. The density is used to detect signature events which are found to be rare with respect to the classification imposed on the signatures. Finally, tests of homogeneity are used to partition the observed signatures, when necessary, into equivalence classes having the same density characterization.

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